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Turbulence transition in shear flows: chaos in high-dimensional spaces

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Summary

Pipe flow, plane Couette flow and boundary layers show turbulent behaviour without a linear instability of the underlying laminar profile. Accordingly, the well established routes to chaos and turbulence through sequences of instabilities that give rise to progressively more complex states cannot apply in their original form since the first step is absent. Experimentally, one finds that the flow rates above which turbulence can be observed are not well characterized and cover a range of values, that turbulence is transient and shows characteristics of a strange saddle rather than a chaotic attractor, and that there is a transition from localized turbulent patches to a spreading phase with spatio-temporal chaotic dynamics. The extension of dynamical systems theories and concepts to high-dimensional spaces has provided the framework in which many of these phenomena can be explained and studied. For recent reviews, see [1, 2, 3]. In my presentation I will survey the ideas that have contributed to our present day understanding of the transition phenomena in pipe flow as and to the possible resolution of this long-standing puzzle surrounding turbulence transition without linear instability.

Critical Reynolds numbers

The linearization of the equations of motion for small deviations from the linear profile gives rise to a non-normal linear operator. Since eigenvectors of non-normal operators are not orthogonal, linear instability does not imply a monotonic decay of perturbations. The dominant physical mechanism are vortices that extract enery from the velocity gradient to built up streaks in the downstream flow component [4]. Within linear theory, both vortices and streaks decay eventually, but the process is capable of transiently increasing the energy content of the perturbation. The energy increase is controlled by the symmetric part of the linearized operator, and its eigenvalues are negative up to some energy stability Reynolds number Re_E. For pipe flow, the value is $Re_E = 81.5$ [5]

The dynamical systems picture of a turbulent state suggests that the turbulence forms around persistent, three-dimensional structures. The first examples of three-dimensional coherent structures were found by continuation from known instabilities in the case of plane Couette flow [6, 7]. In the meantime many of these states were found and in all cases they preceed the observation of turbulence in experiments [8].

Transient turbulence

Much of the variability in the critical Reynolds numbers that are quoted in the literature can be attributed to the fact that even if a turbulent state is realized, it does not persist forever but can decay. Much information is carried in the distribution of lifetimes, which in all cases studied turns out to be exponential, i.e. the probability $P(t)$ to be turbulent at time t varies like $P(t) \sim \exp(-t/\tau(Re))$ [9, 10]. This exponential decay is characteristic of the escape from a strange saddle. The mean lifetime $\tau(Re)$ increases with Reynolds numbers, as is to be expected. According to the most complete studies [11, ?, 12], the lifetimes increase superexponentially, very much like $\tau(Re) \sim \exp(aRe + bRe^2)$. While this quickly becomes very large, it does not diverge at a finite Re, so that there is no transition to a persistent chaotic attractor.

Spatio-temporal aspects

Already Reynolds noted the existence of localized structures in pipe flow, in the form of puffs (at lower Re) and slugs (at higher Re). As the Reynolds number is increased, one finds that puffs can split and spread into the neighboring laminar regions. The fraction F of space covered by turbulence therefore increases with Re. In the limit of infinite system size the transition from localized to spreading turbulence then shows up as a transition from a vanishing value of F to a non-zero one [13, 14].

Edge states

The coexistence of laminar and turbulent flows (even if they are only transient) implies the existence of some boundary between small perturbations that relax to the laminar profile and stronger ones that become turbulent. Using the technical tool of edge state tracking [15, 16, 17], which allows to follow trajectories that neither relaminarize nor become turbulent it has been possible to show that the boundary is formed by the stable manifold of a co-dimension one relative attractor inbetween laminar and turbulent motion. In spatially extended systems, these edge states are localized [18], consistent with the expectation that a localized perturbation should be sufficient to initiate turbulence in the system.

References

[1] Bruno Eckhardt, Tobias M Schneider, Bjorn Hof, and Jerry Westerweel. Turbulence transition in pipe flow. *Annual Review of Fluid Mechanics*, 39:447–468, 2007.

[2] Bruno Eckhardt. Turbulence transition in pipe flow: some open questions. *Nonlinearity*, 21:T1–T11, 2008.

[3] Bruno Eckhardt. Introduction. turbulence transition in pipe flow: 125th anniversary of the publication of reynolds' paper. *Philosophical Transactions Royal Society London A*, 367:449–455, 2009.

[4] S. Grossmann. The onset of shear flow turbulence. *Rev. Mod. Phys.*, 72:603–618, 2000.

[5] D. D. Joseph. *Stability of Fluid Motions, vol I and II.* Springer, 1975.

[6] M. Nagata. Three-dimensional finite-amplitude solutions in plane Couette flow: bifurcation from infinity. *J. Fluid Mech.*, 217:519–527, 1990.

[7] R. M. Clever and F. H. Busse. Tertiary and quaternary solutions for plane Couette flow. *J. Fluid Mech.*, 344:137–153, 1997.

[8] Holger Faisst and Bruno Eckhardt. Traveling waves in pipe flow. *Physical Review Letters*, 91:224502 (4 pages), 2003.

[9] Holger Faisst and Bruno Eckhardt. Sensitive dependence on initial conditions in transition to turbulence in pipe flow. *Journal of Fluid Mechanics*, 504:343–352, 2004.

[10] Tobias M Schneider and Bruno Eckhardt. Lifetime statistics in transitional pipe flow. *Physical Review E*, 78:046310 (10 pages), 2008.

[11] Björn Hof, Jerry Westerweel, Tobias M Schneider, and Bruno Eckhardt. Finite lifetime of turbulence in shear flows. *Nature*, 443:60–64, 2006.

[12] Marc Avila, Ashley P Willis, and Björn Hof. On the transient nature of localized pipe flow turbulence. *Journal of Fluid Mechanics*, 646:127–136, 2010.

[13] Dirk Jan Kuik, C Poelma, and Jerry Westerweel. Quantitative measurement of the lifetime of localized turbulence in pipe flow. *J. Fluid Mech.*, 645:529–539, 2010.

[14] J. C. Rotta. Experimenteller beitrag zur entstehung turbulenter strömung im rohr. *Ing. Archiv*, 24:258–281, 1956.

[15] David Moxey and Dwight Barkley. Distinct large-scale turbulent-laminar states in transitional pipe flow. *Proc. Natl. Acad. Sci. USA*, 107(18):8091–8096, 2010.

[16] Joseph D Skufca, James A Yorke, and Bruno Eckhardt. Edge of chaos in a parallel shear flow. *Physical Review Letters*, 96:174101 (4 pages), 2006.

[17] Tobias M Schneider, Bruno Eckhardt, and James A Yorke. Turbulence transition and the edge of chaos in pipe flow. *Physical Review Letters*, 99:034502, 2007.

[18] Jürgen Vollmer, Tobias M Schneider, and Bruno Eckhardt. Basin boundary, edge of chaos, and edge state in a two-dimensional model. *New Journal of Physics*, 11:013040, 2009.

[19] Fernando Mellibovsky, Alvaro Meseguer, Tobias M Schneider, and Bruno Eckhardt. Transition to localized turbulence in pipe flow. *Physical Review Letters*, 103:054502, 2009.